2026 Pi Day Math Competition
ROOM 1

After entering the factory, they find a cheese vault that seems to be shimmering suspiciously. There is something inscribed on the lock:

"Sdbc rw zdnbx!"

Willa assumes it's a Caesar cipher, where each letter is shifted by some number of letters. For example, a Caesar cipher with a rotation of 1 maps $a \to b$, $p \to q$, etc. and a Caesar cipher with a rotation of 5 maps $t \to y$, $v \to a$.

a. **Decode "upul", given it has a shift of 7 letters.** [2 points]

b. **Decode the message on the lock. "Sdbc rw zdnbx!"** [3 points]

a. Notice that $u \to n$ since u occurs seven letters after n. After using similar logic for the remaining letters, we conclude that "upul" $\to$ "nine". Thus the answer is nine

b. Using the first part as a hint, we conclude that the message might have a shift of nine. Decoding, we find the message to be Just in queso!

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ROOM 2

After opening the vault, Willa and Wallis follow cheese crumbs to their next destination. Unfortunately for Wallis, his brain starts turning into mush as a result of being on the moon too long! He starts to feel faint, and to save him, Willa writes down an infinite fraction with 6s and 7s.

$$6 + \cfrac{7}{6 + \cfrac{7}{6 + \cfrac{7}{6 + \cdots}}}$$

What is the simplified positive value of Willa's number? [6 points]

Let $x = 6 + \frac{7}{6+\frac{7}{6+\dots}}$. Since the fraction is infinite, we know that $x = 6 + \frac{7}{x} \implies x^2 - 6x - 7 = (x-7)(x+1) = 0$. This gives us the solutions $x \in \{-1, 7\}$. Since x is clearly positive, $x = -1$ is extraneous and thus $x = 7$. Thus the answer is $\boxed{7}$

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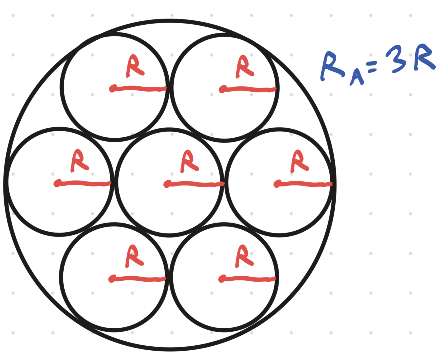
ROOM 3

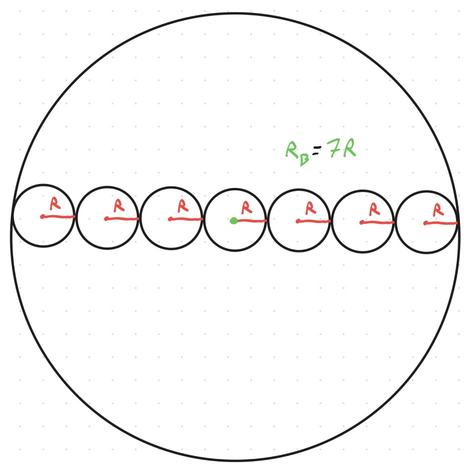
After Wallis recovers, they stumble upon two perfectly circular craters. Seven pies with radius r lie on each crater, and each pie is tangent to at least one other pie. The two craters are described below:

1. Crater A fits the 7 pies in the tightest way possible.
2. Crater B is the largest possible crater that tightly encloses a single connected chain of 7 pies.

Let R_A and R_B be the radii of craters A and B, respectively.
Sketch the arrangement of the pies using 7 smaller circles inside each of the given circles that satisfy the above conditions.

What is R_A/R_B? Express your answer as a common fraction. Use your sketches to help! [8 points]





After drawing, it is somewhat obvious that the ratio between an optimally packed crater and the least optimally packed crater is $\frac{3}{7}$. We can also see this result on Wikipedia. Thus the answer is $\boxed{\frac{3}{7}}$.

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ROOM 4

Oh no! A Moonquake has led to a volcanic eruption. Cheese fondue rains from the sky! Luckily, Willa sees a collection of seven stacked distinct leaves, with colors ranging from across the rainbow.

a. Willa wants to hold one leaf in her left paw, and a different leaf in her right paw. How many ways can she do this? [3 points]

b. Willa grabs a pair of leaves from the pile of 7 to use as umbrellas. Now, order doesn't matter. Example: Grabbing green, then red is the same as grabbing red, then green. **How many distinct pairs of leaves can she choose to take with her?** [3 points]

c. A bug skitters by and is also in need of an umbrella! If Willa picked 3 leaves one by one there would be $7 \times 6 \times 5 = 210$ ways to do this. However, order doesn't matter. **How many different groups of 3 leaves can Willa choose from the pile of 7?** (Hint: divide 210 by the number of ways you can order 3 distinct objects) [3 points]

1. Choosing 1 from 7 distinct leaves, one will have 6 left over. Now she has 6 options. This yields $7 \times 6 = 42$. Note that this is also equal to $\frac{7!}{(7-2)!} = \frac{7!}{5!}$.
2. This is similar to the previous problem, except which leaf is in which hand doesn't matter. This means that our answer is $\frac{42}{2} = \boxed{21}$, since $2! = 2$.
3. This is simply $\binom{7}{3} = \frac{7!}{3!(7-3)!} = 210 \cdot \frac{1}{3!} = \boxed{35}$

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ROOM 5

They leave the pie covered craters and find themselves in a strange habitat with alien species. Wallis discovers the rules of this alien food chain:

1. An alien species will hunt any animal that has **more eyes** OR **fewer feet** than itself.
2. However, if two species both view each other as prey, they call a truce and neither will eat the other!

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a. Species α has 1 eye and 2 feet and species β has 1 eye and 3 feet. **Which species eats the other?** [3 points]

b. Assume an alien species can have anywhere from 1 to 4 eyes and 2 to 8 feet. If Wallis has two eyes and four feet, **what is the ratio of species that will hunt him to species that won't hunt him?** Assume all possible species are present. [4 points]

c. **What is the probability of at least one species hunting him if there are** 2 **species present?** [5 points]

1. Since species α has less feet than species β, species β wants to eat species α. However, they have the same number of eyes so there is no contradiction here. Thus our answer is $\boxed{\beta}$.
2. First we will denote species A be the tuple (e, f), where e is the number of eyes and f is the number of feet. In order for an animal view Wallis as prey, it must satisfy
   - $(e = 1 \text{ AND } f \geq 4) \text{ OR } (e = 2 \text{ AND } f \geq 5)$

   This gives us 9 possible species that would want to eat Wallis. There are $4 \times 7 = 28$ possible species, which gives us an answer of $\frac{9}{28-9} = \boxed{\frac{9}{19}}$
3. Using complementary probability, we want $1 - P(\text{zero predators})$. There are two ways we can calculate this:
   (a) The probability the first species is safe is $\frac{19}{28}$ and the probability of the second species being safe (given the first species is) is $\frac{18}{27}$. Multiplying these together we get $\frac{19 \cdot 18}{28 \cdot 27} = \frac{19}{42}$. This implies $P(\text{zero predators}) = \boxed{\frac{23}{42}}$

(b) We can also calculate $1 - \frac{\binom{19}{2}}{\binom{28}{2}} = 1 - \frac{19}{42} = \frac{23}{42}$

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ROOM 6

After narrowly escaping the aliens, the ground begins to shake. A moonquake!!! A sign rises from the middle of a crater. "Please solve these questions in order to paint the eclipse for King Lunartic".

a. What are the 2 roots of $3x^2 + 5x - 2$? (Hint: One is a fraction) [3 points]

b. What is the sum and the product of the 2 roots you found? [3 points]

c. The equation $2x^2 - 4x - 5$ has solutions p and q. Using Vieta's, find $p^2 + q^2$. Warning: this quadratic is not factorable, so don't solve for p and q directly! [4 points]

a. Here we can simply factor:

$$3x^2 + 5x - 2 = (x + 2)(3x - 1) = 0 \implies \boxed{x \in \left\{-2, \frac{1}{3}\right\}}$$

b. Let Σ_r denote the sum of the roots and Π_r denote the product of the roots. Trivially,

$$\Sigma_r = -2 + \frac{1}{3} = \boxed{-\frac{5}{3}} \text{ and } \Pi_r = -2 \cdot \frac{1}{3} = \boxed{-\frac{2}{3}}$$

c. Using Vieta's formulas, we know that $p + q = -\frac{-4}{2} = 2$ and $pq = \frac{-5}{2}$. Thus $p^2 + q^2 = (p + q)^2 - 2pq = 4 - 2 \cdot \frac{-5}{2} = \boxed{9}$.

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ROOM 7

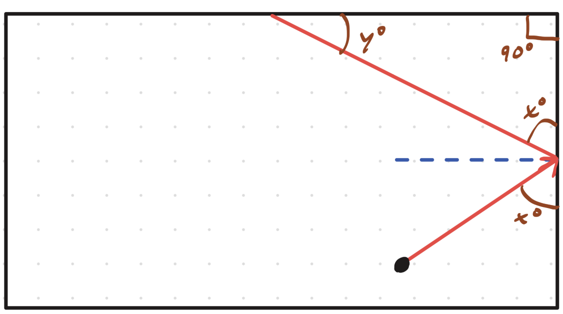
After escaping the alien habitat, they discover the aliens and the cheese fondue volcanoes were an attempt by the ruler of the moon, King Lunartic, to deter them from stealing his relic. King Lunartic, confident in his pool game, states that if they can beat him he will let them walk away with the relic.

Consider a ball on a rectangular pool table. The diagonal of the box is 2 units, and the short side is 1 unit. **Every time the ball hits a wall, its path gets reflected about the line perpendicular to the wall**.

a. If the ball first bounces on the right wall with an angle x° and then the top wall, **what will the ball's angle be as it is leaving the top wall? Express your answer in terms of x°.** [5 points]

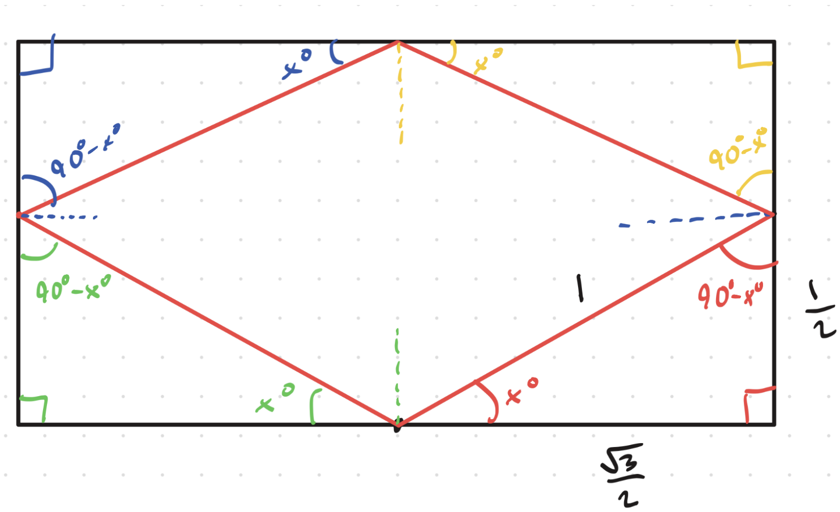
b. Now imagine the ball starts on the midpoint of the bottom wall with an angle y°. **What must y° be if the ball bounces off every wall exactly once and returns to its starting point?** [10 points]

1. Consider the following diagram:



Observing this, we notice that a right triangle is formed. Since Euclidean Triangles have angles that add up to 180°, we know that $x + 90 + y = 180 \implies y^\circ = \boxed{90^\circ - x^\circ}$

2. Given that the diagonal has length 2 and the shorter side has length 1, we can use the Pythagorean theorem to find the length of the longer side, which is $\sqrt{2^2 - 1^2} = \sqrt{3}$. Now consider the following diagram:



This gives us a right triangle with side length ratios $1 : \sqrt{3} : 2$ which matches our hint perfectly (after we divide by 2), giving us $y^\circ = \boxed{30^\circ}$

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ROOM 8

Willa and Wallis are finally ready to leave. However, first they must make their way to their ship. To their horror, they find that their oxygen tanks are depleting fast. However, running faster depletes oxygen at a faster rate.

If their amount of oxygen loss by the time they get to the ship is $s+\frac{9}{s}$, where s is their speed, **what is the speed in which they reach their destination with the least amount of oxygen loss? Assume s is constant.** [10 points]

We want to minimize $s+\frac{9}{s}$. Here we can use AM-GM: Letting $x=s$ and $y=\frac{9}{s}$, $\frac{s+\frac{9}{s}}{2} \geq \sqrt{\frac{9}{s}\cdot s} = 3$. This means that $s+\frac{9}{s} \geq 6$. Equality is satisfied when $s+\frac{9}{s}=6$ which is when $s=3$. Our constant speed is $\boxed{s = 3\text{ft/s}}$