2026 Pi Day Math Competition

Part I

Time: 25 Minutes — Calculators: No

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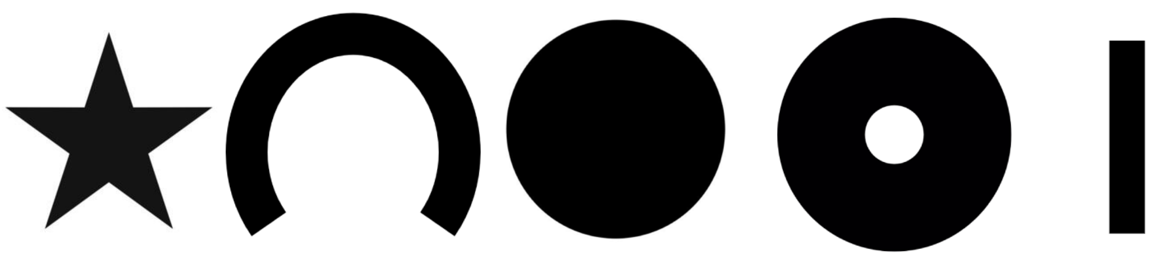
1. If 5 consecutive integers sum to -5, what is the first integer?

Let x be our smallest number. Then $x + (x+1) + (x+2) + (x+3) + (x+4) = 5x + 10 = -5$. This gives us $5x = -15$ which gives us $x = \boxed{-3}$.

2. If a circular wheel with a radius of 5 cm does 3 full rotations, how far does it travel?

Remembering that the circumference of a circle is $2\pi r$, where r is the radius, the circumference of the wheel is 10π cm. This means that after 3 rotations the total distance traveled will be $\boxed{30\pi \text{ cm}}$.

3. We call a shape star-shaped if there exists a point a inside it such that we can draw straight lines from a to every other point in the shape, each contained within the shape. Which of the following are star-shaped? (Circle them on your answer sheet)



Trivially the first, third, and fifth shapes.

4. There are 100 astronauts at the ISS. 67 of them have a Special Water Tube to drink from. 41 of them have a Special Lunch Pipe to eat from. 21 of them have both. How many have neither?

Let W be the number of astronauts that ONLY have a Special Water Tube and L be the number of astronauts that ONLY have a Special Lunch Pipe. $W = 67 - 21 = 46$ since 21 of the astronauts with a Special Water Tube also have a Special Lunch Pipe. Applying the same logic, $L = 41 - 21 = 20$. Since $W + L + B + N = 10$, where B are astronauts with both and N represents the number of astronauts with neither, we notice that $46 + 20 + 21 + N = 100 \implies N = \boxed{13}$

5. A *googly* number is a number where the product of its digits is prime. How many *googly* numbers are there less than 1000?

First, remember that by the definition of a prime number, $a \cdot b$ is not prime, unless a is prime and b is 1, or vice versa. Let $p \in \{2, 3, 5, 7\}$ (the prime numbers less than ten). Notice that our googly number, g, has to be in one of the following forms:

1. $p11$
2. $1p1$
3. $11p$
4. $p1$
5. $1p$
6. p

Since there are four such ps and 6 such possibilities, the total number of possibilities is $6 \cdot 4 = \boxed{24}$.

6. In the world championship slapping competition, each competitor slaps every other competitor once. If there are 42 slaps, how many people are in the world championship?

Let n be the number of people in the slapping competition. This means that each competitor has $(n-1)$ slaps. Since there are n competitors, the total number of snaps is $n(n-1) = 42$ which gives us the quadratic equation $n^2 - n - 42 = 0$. This gives us $n = \boxed{7}$.

7. A team of six is running a piggy-back race:

1. Damian has a running speed of 10 m/s.
2. Lisa has a running speed of 8 m/s.
3. Kareem has a running speed of 12 m/s.
4. Kareena has a running speed of 14 m/s.
5. Nicholas has a running speed of 14 m/s
6. Wiley has a running speed of 12 m/s.

In this particular race, each person partners up with someone else who they carry on their back. Their "score" is their total distance (across all three pairs) they can run in 10 seconds, which they seek to maximize. Assume that each runner's running speed is unaffected by having someone on their back.

a. Find the optimized score
b. Find the total amount of player combinations that result in this optimized score

We simply want the fastest runners running, which is Kareena, Nicholas, and Wiley OR Kareem. This means that their optimized score is $10 \cdot (14 + 14 + 12) = \boxed{400}$. Since there are two choices for runners, and 3! choices for riders after a choice of runners, we have $2 \cdot 3! = \boxed{12}$ different combinations.

8. The Sonoma County student council has 3 Maria Carrillo graduates and 3 Tech High graduates. If we randomly select two of them to co-chair a committee, what is the probability that these chairpersons are graduates from the same high school? Express your answer as a simplified fraction.

There are two ways to solve this problem

1. First, WLOG, choose a student. That student goes to school α. If we are to choose another student randomly, the chances of that second student being from the same school is $\boxed{\frac{2}{5}}$.

9. What is the value of

$$\left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \left\lfloor \frac{2}{2} \right\rfloor + \left\lceil \frac{2}{2} \right\rceil + \left\lfloor \frac{3}{2} \right\rfloor + \left\lceil \frac{3}{2} \right\rceil + ... + \left\lfloor \frac{31}{2} \right\rfloor + \left\lceil \frac{31}{2} \right\rceil ?$$

We can quickly realize a pattern...

$$\left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \left\lfloor \frac{2}{2} \right\rfloor + \left\lceil \frac{2}{2} \right\rceil + \left\lfloor \frac{3}{2} \right\rfloor + \left\lceil \frac{3}{2} \right\rceil + ... + \left\lfloor \frac{31}{2} \right\rfloor + \left\lceil \frac{31}{2} \right\rceil$$
$$= 0 + (1+1+1+1) + (2+2+2+2) + \cdots + (15+15+15+15) + 16$$
$$= 4 \cdot \sum_{i=1}^{15} i + 16 = 480 + 16 = \boxed{496}$$

10. Find the sum of the units digits (1s place digits) of the numbers

$$1!^2,\ 2!^2,\ 3!^2,\ \ldots,\ 314!^2.$$

Since $5!, \ldots 314!$ all contain factors of 2 and 5, and thus 10, we know that they have a ones digit of 0. Thus we only need to consider $1!^2, 2!^2, 3!^2$, and $4!^2$. $1!^2 = 1$ which has a ones digit of 1, $2!^2 = 2^2 = 4$ which has a ones digit of 4, $3!^2 = 6^2 = 36$ which has a ones digit of 6, and $4!^2 = 24^2$ which has a ones digit of 6. If we sum up the ones digits we get $1 + 4 + 6 + 6 = \boxed{17}$

Stop. End of Section One.

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Part II

Time: 25 Minutes — Calculators: No

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1. Bad Bunny has a rectangular box of cookies. One face has an area of 24 square inches. Another face has an area of 30 square inches. The third face has an area of 20 square inches. What is the volume of the cookie container?

Since $V = \ell \times w \times h$, we first want to find those values. Let $24 = \ell \times w$, $30 = w \times h$, and $20 = h \times \ell$. If we substitute, we find that $24 = \frac{20}{h} \times w = \frac{20}{30/w} \times w \implies w = \sqrt{24 * 30/20} = 6$. This gives us $\ell = 4$ and $h = 5$, and thus $V = \boxed{120}$

2. What is the smallest odd number that is a multiple of four different prime numbers?

The four smallest odd prime numbers are $3, 5, 7, 11$. If we multiply these together we get $\boxed{1155}$.

3. Playboy Carti is able to chug matcha at a rate of 1 L/min. However, after each minute of chugging, her chugging rate gets cut in half. If she chugged until the end of time, how many liters would she have chugged?

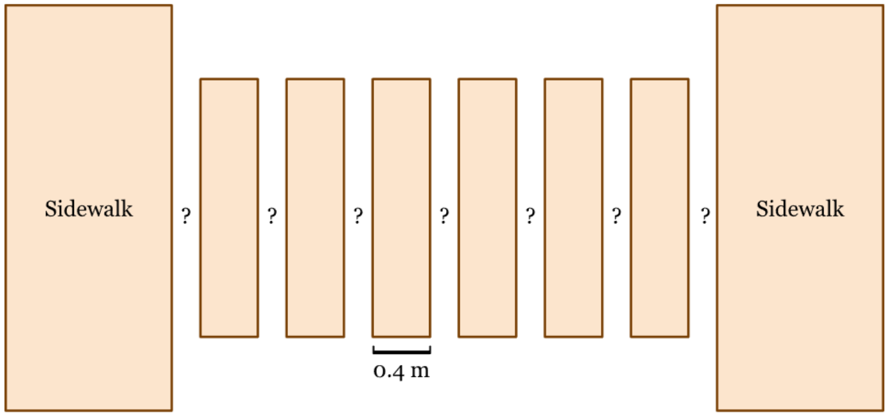
This forms the infinite series

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = \boxed{2}$$

4. Tennis balls are placed on the ground in a filled-in square, then in each space between four tennis balls, another ball is placed, forming another square, finishing with a single ball at the top. If the base is 6x6, how many balls are in the pyramid?

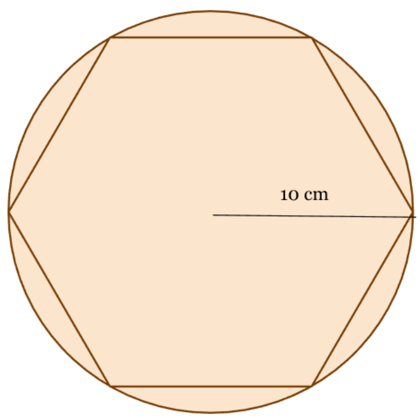
Drawing them out, one will realize that if a layer is $n \times n$, then the layer on top is $(n-1)\times(n-1)$. This means that if the bottom layer is 6×6, which has 36 balls, the layer on top has $5 \times 5 = 25$ balls. This will continue until the top layer has 1 ball. Thus the total number of balls is $36 + 25 + 16 + 9 + 4 + 1 = \boxed{91}$

5. Olivia Rodrigo is painting crosswalk lines. If the sidewalks are 32 meters apart, and each of the 35 equally spaced painted rectangles is 0.4 meters wide, what is the width of each space? There is a space between the sidewalk and the first and last painted rectangles.



The total amount of space taken by the painted rectangles is $0.4 \cdot 35 = 14$ meters. This means that the remaining space is $32 - 14 = 18$ meters. Since there are 36 of these spaces, the space that one space takes up is $\frac{18}{36} = \boxed{\frac{1}{2} \text{ meter}}$

6. If the radius of the circle above is 10 cm, what is the area of the inscribed regular hexagon (the sides of a regular hexagon all have the same length)?



We note that that this hexagon is simply the sum of of 6 equilateral triangles with side-length 10. This is $\frac{\sqrt{3}}{4} \cdot 100 \cdot 6 = \boxed{150\sqrt{3}}$

7. Let a rectangle ABCD exist in the coordinate plane such that:

- A is at (-1,-1)
- B is at (-1,3)
- C is at (5,3)

- D is at (5,-1)

What is the ratio of the area of ABCD and the self-intersecting polygon ACBD? (To form ACBD, connect the vertices in the order of $A \to C \to B \to D \to A$)

Just draw and you will see it is 2.

8. Ronaldo is pretty good at soccer. The probability that he makes a goal is unknown, let it be p. He takes 5 kicks in a row. The ratio of the probability that he makes 2 goals to the probability that he makes 3 goals is $\frac{1}{2}$. What is p?

The probability of Ronaldo making three goals, irrespective of order is $p^3(1-p)^2 \cdot \binom{5}{3} = 10p^3(1-p)^2$ and the probability of him making three goals is $\binom{5}{2}p^2(1-p)^3$. Thus $P_2/P_3 = \frac{1-p}{p} = \frac{1}{2} \implies p = \boxed{\frac{2}{3}}$

9. Lisa really, really likes sprinkles, and she has a LOT of them. When she tries to sort them into piles of 3, she has 2 left over. When she tries to sort them into piles of 4, she has 3 left over. When she tries to sort them into piles of 5, she has 4 left over. What is the smallest amount of sprinkles Lisa could have?

Let n be the number of sprinkles. We can form the following system:

$$\begin{cases} n \equiv -1 \mod 3 \\ n \equiv -1 \mod 4 \\ n \equiv -1 \mod 5 \end{cases}$$

We can also express this number n in the following way (or we can use chinese remainder theorem):

Since $n = 3k_1 - 1$ and $n = 4k_2 - 1$, $k_i \in \mathbb{N}$ we know that $n = 12k_2 - 1$. Applying similar logic, $n = 60k_3 - 1$. This gives us $n = \boxed{59}$.

10. Shengkai decides to start a farm where he has a field with 50 female cows and 50 male cows, for a total of 100. If Shengkai pairs each cow with another cow such that there are 50 pairs of cows, what is the expected number of pairs of cows that consist of one male cow and one female cow?

The total number of possible combinations is $\binom{100}{2}$ and the total number of male female combinations is $\binom{50}{1} \times \binom{50}{1}$. Since each probability is equally likely to appear, the probably of a pairing being male-female is

$$\frac{\binom{50}{1} \times \binom{50}{1}}{\binom{100}{2}} = \frac{2500}{4950}$$

Since this scales linearly, our expected number of cows is

$$50 \cdot \frac{2500}{4950} = 50 \cdot \frac{50}{99} = \boxed{\frac{2500}{99}} = \boxed{25.\overline{25}}$$