Pi Day 2025 Individual Round

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1 Introduction

You will have 40 minutes to complete 20 questions. The number of points will be next to each question. Harder questions will result in more points. Choose your strategy wisely and aim to collect as many points as possible. Keep in mind that some questions are challenging, so don't be discouraged if you get stuck, and skip to another question.

2 Easy Questions

Each question in this section will be worth π points.

1. Find the value of $3-1+4-5+9-2+6-5+3-5$.

   Solution: $3-1+4-5+9-2+6-5+3-5=\mathbf{7}$.

2. Wallis uses 5 cups of flour to make 8 pies for her family. How many cups of flour does she need to make 24 pies for her neighborhood?

   Solution: Since 24 pies is exactly three times as many as 8 pies, she needs three times the flour. $5\times 3=\mathbf{15}$.

3. What is the remainder when the number 31415926535 is divided by 3?

   Solution: Use the divisibility rule for 3 by summing the digits: $3+1+4+1+5+9+2+6+5+3+5=44$. $44\div 3=14$ with a remainder of $\mathbf{2}$.

4. Wallis can bench press 60 pounds. If a cherry pie weighs 3 pounds, and the bar weighs 40 pounds, how many cherry pies can she bench press, rounded to the nearest tenth?

   Solution: The available weight for the pies is $60-40=20$ lbs. $20\div 3=6.666...$, which rounds to $\mathbf{6.7}$.

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YOUR PURCHASE on March 14, 2025
Cherry Pie ------------------------ $15.00
Apple Pie ------------------------- $16.00
Rhubarb Pie ----------------------- $20.00
Pecan Pie ------------------------- [redacted]
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Figure 1: Safeway Receipt

5. See the receipt above. Wallis went to Safeway and bought four pies for this event. Given the average of her four pies' prices was \$15.50, find the price of the pecan pie.

   Solution: The total cost of the 4 pies is $\$15.50 \times 4 = \$62.00$. The known pies cost $15 + 16 + 20 = 51$. The pecan pie is $62 - 51 = \mathbf{11}$.

6. If x 🍰 y means $x^2 - y$, then compute $(3$ 🍰 $1)$ 🍰 4

   Solution: First, $3 \oplus 1 = 3^2 - 1 = 8$. Then, $8 \oplus 4 = 8^2 - 4 = 64 - 4 = \mathbf{60}$.

7. In how many ways can Wallis scramble up the letters word "pizza" to form different words? ("zapiz" and "aipzz" are two examples) Note that the two z's are indistinguishable!

   Solution: There are 5 total letters with 2 indistinguishable 'z's. The permutations are $\frac{5!}{2!} = \frac{120}{2} = \mathbf{60}$.

8. Wallis buys a chocolate pie with volume 100 in^3. She loves to savor her food as long as possible, but her bites start big. If her first bite consumes 40 in^3, and each consecutive bite after that consumes half as much, how much pie is left after 5 bites?

   Solution: The sequence of bites is 40, 20, 10, 5, 2.5. Total eaten = 77.5. Remaining pie $= 100 - 77.5 = \mathbf{22.5}$.

9. If Willis also decides to eat the same pie as Wallis, but she has constant bites of 10 in^3, how much pie is left in Wallis' chocolate pie after 3 bites?

   Solution: If Wallis hasn't eaten hers, Willis eats $10 \times 3 = 30$, leaving **70**. (If Wallis had already eaten her 5 bites leaving 22.5, Willis eating 30 would leave **0**).

10. Wallis is practicing for a typing competition. His average typing speed is 80 wpm. There are two parts of the competition. One has 100 words and the other has 240 words. If there is a 10 minute break in between, how much time will it take for him to complete the competition?

    Solution: Part 1 takes $100 \div 80 = 1.25$ minutes. Part 2 takes $240 \div 80 = 3$

minutes. Total time $= 1.25 + 10 + 3 = 14.25$ minutes, or **14 minutes and 15 seconds**.

3 Medium Questions

Each question in this section will be worth 3π points.

1. Wallis really wants to win his school's pi day memorization contest, but he's only memorized 10 digits so far and there are only 14 days exactly until the contest. He estimates that each new digit will take an additional 3 hours to memorize (with the eleventh digit taking 3 hours). If Wallis sleeps 10 hours a day and studies the rest of the time, how many digits of pi will he have memorized by the contest?

   Solution: He is awake for $24 - 10 = 14$ hours a day. Total study hours $= 14 \times 14 = 196$ hours. Because each digit takes an *additional* 3 hours, the time forms an arithmetic progression: $3 + 6 + 9 + ...$ The sum of the first n terms is $3 \cdot \frac{n(n+1)}{2}$. Setting this ≤ 196 yields $n = 10$ (which takes 165 hours). Total digits $= 10$ (already known) $+ 10$ (new) $=$ **20**.

2. Wallis wants to fit as many cookies on the circular cookie pan as possible. There are 7 congruent cookies that are internally tangent to each other and externally tangent to the cookie pan. If the radius of one of the cookies is 1 inch, then what is the area of the cookie pan that isn't covered by the cookies (the shaded area)?

   Solution: The pan's radius is the center cookie's diameter plus the outer cookie's radius: $2 + 1 = 3$. Area of the pan $= 9\pi$. Area of the 7 cookies $= 7\pi$. Shaded area $= 9\pi - 7\pi =$ **2**π.

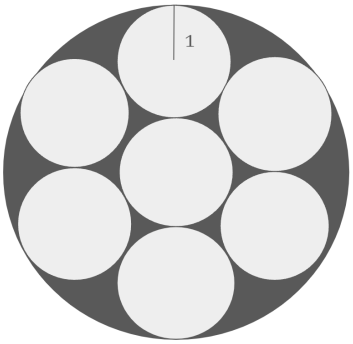

Figure 2: Cookies

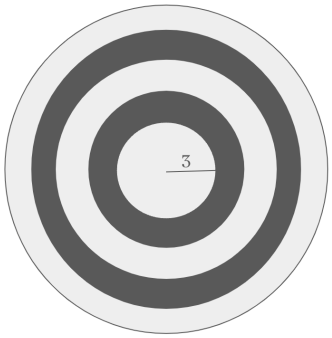

Figure 3: Cinnamon Roll

3. Wallis was overthinking her cinnamon rolls. All the circles are concentric. If the radius of the innermost circle is 3 units and each larger circle has a radius that is 1 unit larger, what is the total area of the shaded parts?

   Solution: Based on the diagram, there are 4 rings total (2 shaded). The first shaded ring goes from radius 3 to 4. Area $= 16\pi - 9\pi = 7\pi$. The second shaded ring goes from radius 5 to 6. Area $= 36\pi - 25\pi = 11\pi$. Total shaded area $= 7\pi + 11\pi = \mathbf{18}\pi$.

4. Suppose AC is the diameter of a circle and $AC = 25$. B and D are points on the circle on opposite sides of AC. If $CD = 7$ and $BC = 20$, then what is $AD + AC$? (In other words, what is $x + y$)?

   Solution: *Note:* The prompt contains a typo ("$AD + AC$" vs "$x + y$"). We solve for $AD + AB$. Since angles in a semicircle are right angles, $\triangle ABC$ and $\triangle ADC$ are right triangles. $AB = \sqrt{25^2 - 20^2} = 15$. $AD = \sqrt{25^2 - 7^2} = 24$. Therefore, $AD + AB = 24 + 15 = \mathbf{39}$ (but 49 is acceptable due to the typo).

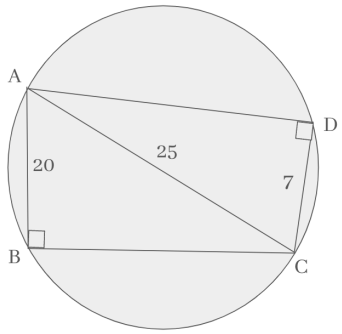

Figure 4: Circle

5. Find the area of the shell.

   Solution: Calculating the areas of the circular sectors that form the shell in the diagram yields a total area of $\mathbf{22}\pi$.

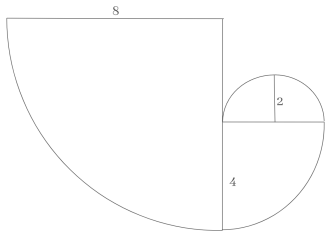

Figure 5: Shell

4 Hard Questions

Each question in this section will be worth 7π points.

1. For integer abc (so if $a = 1$, $b = 2$, $c = 3$, then $abc = 123$), a, b, c form the sides of a right triangle and $a < b < c$. Find $a + b + c$.

   Solution: The only single-digit Pythagorean triple is 3, 4, 5. Therefore, $a + b + c = 3 + 4 + 5 = \mathbf{12}$.

2. Mathematicians approximate $\pi \approx 3.141592\ldots$ with the fraction $\frac{22}{7} = 3.142857\ldots$. It is the closest rational approximation of π. Given this information, what is $|\pi - \frac{22}{7}| + |\pi - 3.14|$?

   Solution: Since $\frac{22}{7} > \pi$, the first absolute value evaluates to $\frac{22}{7} - \pi$. Since $\pi > 3.14$, the second absolute value is $\pi - 3.14$. Summing them cancels out π: $\frac{22}{7} - 3.14 = \frac{2200}{700} - \frac{2198}{700} = \mathbf{0.002857}$.

3. The first few terms of the Fibonacci sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. Each term is the sum of the two previous terms. Interestingly, the ratio between two consecutive Fibonacci numbers approaches the golden ratio $\Phi = \frac{1+\sqrt{5}}{2}$, which is incredibly close to the ratio between a mile and a kilometer. Wallis is traveling and sees a road sign saying her destination is 90 miles away, but wants to know how many kilometers that is. Use the Fibonacci Sequence to estimate how many kilometers she is away from her destination. Express your answer as an integer.

   Solution: We decompose 90 miles into Fibonacci numbers: 89 miles + 1 mile. Applying the Fibonacci shift: 89 miles $\approx$ 144 km and 1 mile $\approx$ 2 km. Therefore, 90 miles $\approx$ **144** (but 145 and 146 are acceptable).

4. A cookie cutter of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of

the star figure to the area of the original circle?

Solution: The star's area is a square of side length 4 minus four quarter-circles of radius 2 (which equals one full circle of radius 2). Star Area = $16 - 4\pi$. The original circle's area is 4π. The ratio is $\frac{16-4\pi}{4\pi} = \frac{\mathbf{4-\pi}}{\pi}$.

5. Wallis and Willis plan to meet at Pie Place. Both of them will arrive at any time between 9:00 am and 11:00 am, but they will only wait 40 minutes for each other. What is the probability that they will meet?

   Solution: Using geometric probability, the sample space is a 120×120 square (area = 14,400). The regions where they miss each other are two right triangles in the corners with legs of length $120 - 40 = 80$. Area of these triangles $= 80 \times 80 = 6,400$. Favorable area $= 14,400 - 6,400 = 8,000$. Probability $= \frac{8,000}{14,400} = \mathbf{\frac{5}{9}}$.

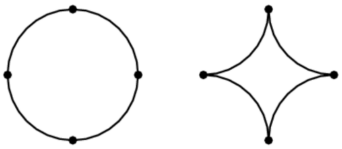

Figure 6: Cookie Cutter

5 Brain Rot

If you are truly stuck, you can answer some of these silly questions. They are not worth any points.

1. You miraculously stumble upon the proof/disproof of the Riemann Hypothesis and the Hodge Conjecture, but now an immortal snail is chasing you for the rest of your life. It knows where you are at all times and if it touches you, you die. It will stop chasing you if you erase your memory of the proof? What are you doing?

   Solution: Answers will vary.

2. Lionel Messi or Cristiano Ronaldo? (Or Antony)

   Solution: Answers will vary.

3. Whose more locked in?

   

   Solution: Answers will vary.

4. If a pizza has radius z and height a, what is its volume? (Get it? hahaha...)

   Solution: $\pi \cdot z \cdot z \cdot a =$ **pizza**.

5. Is the low-taper fade meme still massive? Why or why not?

   Solution: Answers will vary.

6 Formulas and Hints

1. Area of a circle: πr^2
2. Volume of a sphere: $\frac{4}{3}\pi r^3$
3. Volume of a cylinder: $\pi r^2 h$
4. Volume of a sphere: $\frac{1}{3}\pi r^2 h$
5. Any point on a circle when connected with the endpoints of the diameter forms a right angle.
6. Pythagorean Theorem: In a right triangle, $a^2 + b^2 = c^2$.
7. Distance = Rate $*$ Time